Measurement of rotation axis

for asteroid

(1036) Ganymed

Method proposed by
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Change log

Version	Date	Author	Changes
1.0	17-July-2011	J. CARON	First version
1.1	12-Oct-2011	J. CARON	No changes (creation of document template)

Contents

1. Introduction

1.1 Original idea

The following message was posted on AUDE-L Yahoo group (14 June 2011), in French:

> « The asteroid 1036 Ganymed looks like an interesting target for the coming months :it will pass close to Earth, at 0.360 AU in next October.It will also always stay at high declinations (from +10 to +65 deg) with a low magnitude (8 to 11). Besides, MPC is encouraging astrometry on it.
>
> According to JPL Horizons, the rotation period is 10.31 hours which can be reasonnably measured while being not too fast. In the first part of its orbit (June-July) the motion is from 1.8 to 1.3 arcsec per minute in azimuth 30deg. Then in the second part (end of Sept, October, beginning Nov) the motion is from 2 to 3 arcsec per minute, in the opposite direction (azimuth 170 deg) !!
>
> When we measure a rotation period with photometry, I think that the measurement also contains a contribution from the asteroid motion over the sky. The information that we are looking for is the inertial rotation period, while we are measuring the rotation with respect to the line of sight from Earth to asteroid. When we neglect the difference, we neglect the rotation of this line of sight in the inertial frame (which is exactly the asteroid motion over the sky).For 1036 Ganymed, if I am not wrong, the combination of a slow rotation (T=10.31h gives 0.6094 rad/h) and a large change of motion over the sky (+/-2 arcsec per minute gives +/- 0.0006 rad/h) gives a relative importance of 10^-3.

If all this is correct, and if the accuracies over the rotation periods derived by Raoul Behrend are realist (we often get better than 10^-5) we could try to measure the term linked to the asteroid motion. Then, taking the difference between the periods measured in June-July and October, we could determinate whether the rotation is prograde or retrograde.What do you think ?

f this is confirmed, it would be nice to involve a few people to do it (to be sure to have the right coverage in time
due to weather, and to improve the measurements). »

1.2 Ephemeris from MPC

Minor Planet Ephemeris Service: Query Results

Below are the results of your request from the Minor Planet Center's Minor Planet Ephemeris Service. Ephemerides are for the geocenter.

(1036) Ganymed

Display all designations for this object / Show naming citation / # of variant orbits available = 3

Perturbed ephemeris below is based on 46-opp elements from *MPC* 32295. Last observed on 2011 July 10.

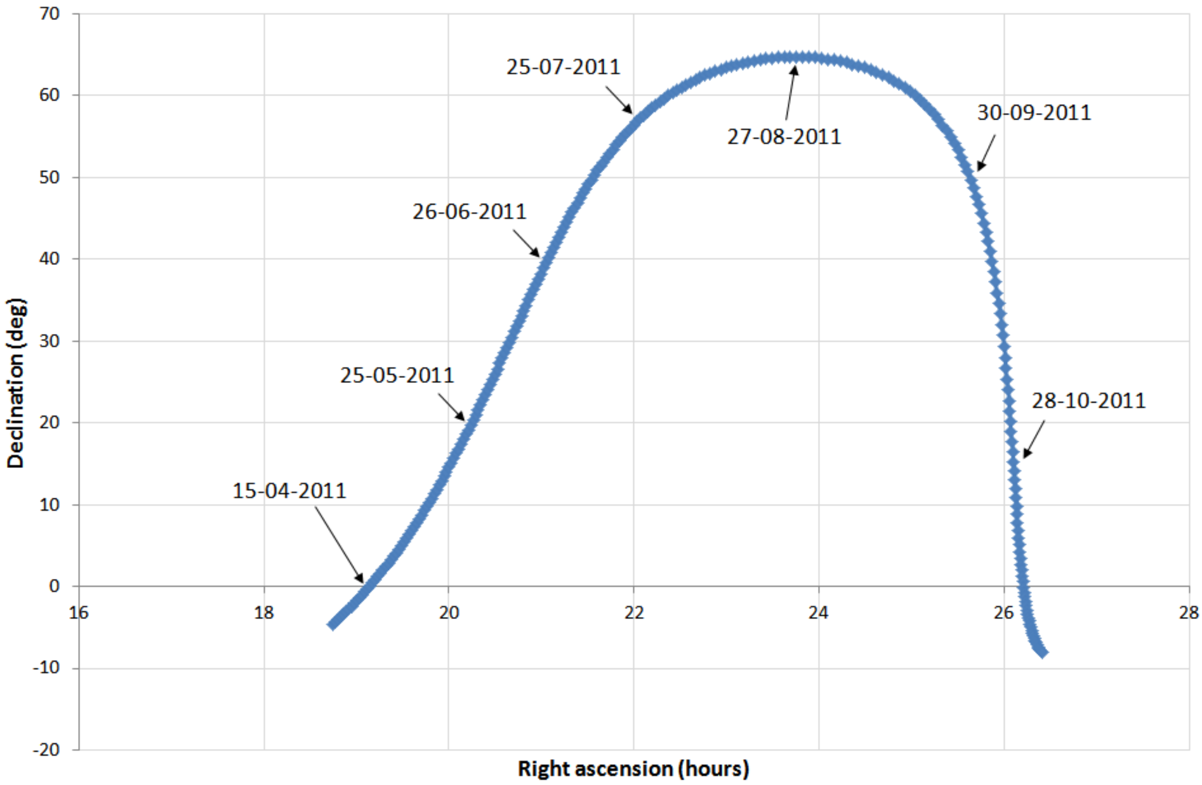
Discovery date : 1924 10 23
Discovery site : Bergedorf
Discoverer(s) : Baade, W.

Further observations? NEO : None needed at this time.

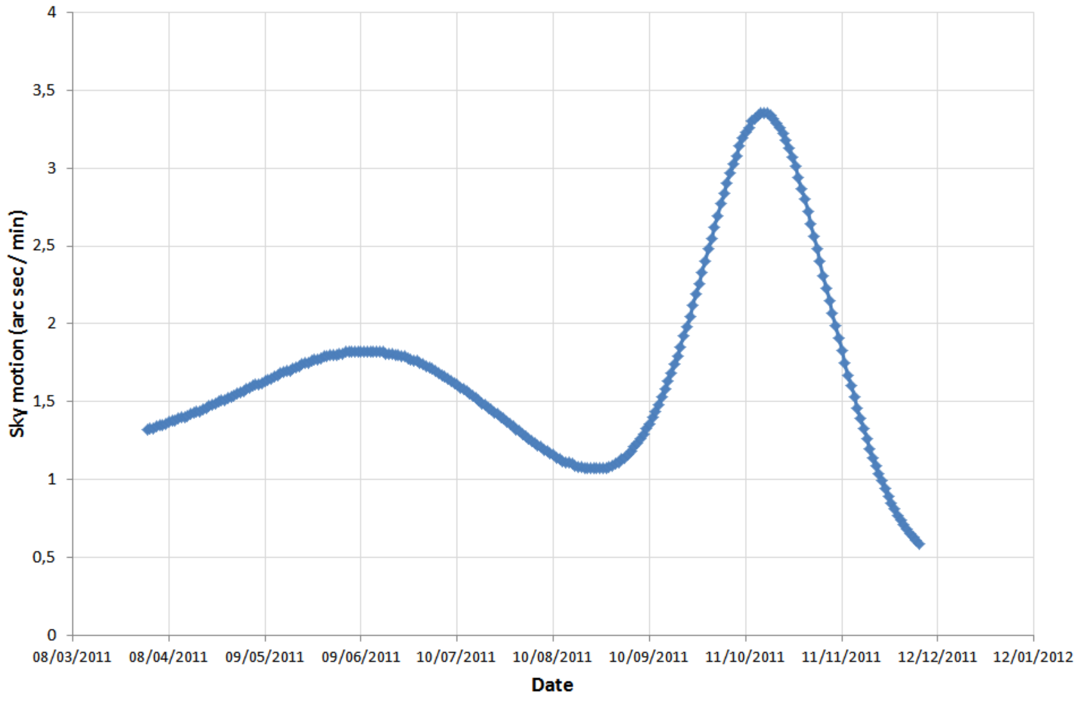
01036

Date	UT (h m s)	R.A. (J2000)	Decl.	Delta	r	El.	Ph.	V	Sky Motion "/min	Sky Motion P.A.	Uncertainty info 3-sig/"	Uncertainty info P.A.	
2011 04 01	000000	18 44 26.7	-04 36 43	1.740	1.993	89.1	30.1	13.2	1.32	055.8	0	338.0	/ Map / Offsets
2011 04 07	000000	18 54 55.3	-02 43 56	1.644	1.953	91.9	30.8	13.1	1.36	052.8	0	336.9	/ Map / Offsets
2011 04 13	000000	19 05 16.7	-00 39 23	1.551	1.913	94.6	31.5	12.9	1.40	049.8	0	335.8	/ Map / Offsets
2011 04 19	000000	19 15 31.0	+01 37 28	1.462	1.873	97.1	32.1	12.8	1.45	046.8	0	334.5	/ Map / Offsets
2011 04 25	000000	19 25 37.9	+04 07 06	1.377	1.833	99.4	32.8	12.6	1.51	043.9	0	333.1	/ Map / Offsets
2011 05 01	000000	19 35 36.7	+06 49 50	1.297	1.794	101.5	33.4	12.4	1.56	041.1	0	331.6	/ Map / Offsets
2011 05 07	000000	19 45 26.9	+09 45 37	1.222	1.754	103.3	34.0	12.3	1.61	038.4	0	329.8	/ Map / Offsets
2011 05 13	000000	19 55 08.8	+12 54 09	1.151	1.715	104.8	34.7	12.1	1.67	035.9	0	327.6	/ Map / Offsets
2011 05 19	000000	20 04 43.8	+16 14 47	1.086	1.676	106.0	35.5	12.0	1.72	033.6	0	325.1	/ Map / Offsets
2011 05 25	000000	20 14 12.7	+19 46 29	1.027	1.637	106.8	36.3	11.8	1.77	031.6	0	322.1	/ Map / Offsets
2011 05 31	000000	20 23 36.3	+23 27 33	0.973	1.600	107.3	37.2	11.7	1.80	029.8	0	318.6	/ Map / Offsets
2011 06 06	000000	20 32 56.3	+27 15 41	0.924	1.563	107.4	38.3	11.5	1.82	028.3	0	314.1	/ Map / Offsets
2011 06 12	000000	20 42 16.4	+31 08 05	0.880	1.527	107.1	39.5	11.4	1.82	027.2	0	308.9	/ Map / Offsets
2011 06 18	000000	20 51 41.5	+35 01 48	0.841	1.492	106.6	40.7	11.3	1.81	026.5	0	302.5	/ Map / Offsets
2011 06 24	000000	21 01 16.8	+38 53 43	0.806	1.458	105.8	42.1	11.2	1.78	026.2	0	295.3	/ Map / Offsets
2011 06 30	000000	21 11 08.3	+42 40 22	0.774	1.427	104.8	43.6	11.1	1.73	026.4	0	287.2	/ Map / Offsets
2011 07 06	000000	21 21 25.2	+46 18 19	0.746	1.396	103.7	45.0	11.0	1.66	027.2	0	278.7	/ Map / Offsets
2011 07 12	000000	21 32 20.5	+49 44 32	0.719	1.368	102.7	46.5	10.9	1.58	028.8	0	090.0	/ Map / Offsets
2011 07 18	000000	21 44 09.7	+52 56 27	0.694	1.342	101.7	47.9	10.8	1.49	031.3	0	082.3	/ Map / Offsets
2011 07 24	000000	21 57 09.0	+55 51 41	0.670	1.318	100.9	49.2	10.7	1.40	034.7	0	075.0	/ Map / Offsets
2011 07 30	000000	22 11 36.3	+58 27 32	0.646	1.298	100.3	50.3	10.7	1.31	039.6	0	069.2	/ Map / Offsets
2011 08 05	000000	22 27 51.4	+60 41 08	0.622	1.280	100.1	51.3	10.6	1.22	046.2	0	064.2	/ Map / Offsets
2011 08 11	000000	22 46 13.4	+62 29 25	0.598	1.265	100.3	52.0	10.5	1.14	055.0	0	060.0	/ Map / Offsets
2011 08 17	000000	23 06 49.8	+63 48 55	0.572	1.254	100.9	52.5	10.4	1.09	066.1	0	056.6	/ Map / Offsets
2011 08 23	000000	23 29 27.3	+64 35 01	0.546	1.246	102.1	52.5	10.3	1.07	079.7	0	053.4	/ Map / Offsets
2011 08 29	000000	23 53 27.1	+64 41 56	0.518	1.242	104.0	52.1	10.1	1.09	095.4	0	050.4	/ Map / Offsets
2011 09 04	000000	00 17 46.5	+64 03 05	0.490	1.241	106.6	51.2	10.0	1.18	112.0	0	046.9	/ Map / Offsets
2011 09 10	000000	00 41 08.6	+62 31 46	0.462	1.244	110.1	49.5	9.8	1.36	127.6	0	042.8	/ Map / Offsets
2011 09 16	000000	01 02 16.3	+60 01 15	0.435	1.251	114.7	46.9	9.6	1.63	141.1	0	038.2	/ Map / Offsets
2011 09 22	000000	01 20 12.6	+56 24 39	0.409	1.261	120.4	43.3	9.4	1.98	152.0	0	033.2	/ Map / Offsets
2011 09 28	000000	01 34 33.3	+51 36 10	0.387	1.275	127.5	38.6	9.2	2.40	160.2	0	028.4	/ Map / Offsets
2011 10 04	000000	01 45 28.0	+45 34 35	0.370	1.292	135.9	32.6	9.0	2.84	166.1	0	024.1	/ Map / Offsets
2011 10 10	000000	01 53 27.1	+38 28 41	0.360	1.312	145.7	25.4	8.8	3.19	170.1	0	020.5	/ Map / Offsets
2011 10 16	000000	01 59 07.5	+30 40 45	0.360	1.335	156.3	17.5	8.6	3.35	172.7	0	017.7	/ Map / Offsets
2011 10 22	000000	02 03 06.8	+22 44 51	0.372	1.360	167.2	9.3	8.5	3.26	174.3	0	015.7	/ Map / Offsets
2011 10 28	000000	02 06 01.1	+15 17 53	0.394	1.388	177.6	1.8	8.3	2.94	175.0	0	014.3	/ Map / Offsets
2011 11 03	000000	02 08 24.0	+08 48 04	0.428	1.418	171.9	5.7	8.7	2.48	174.6	0	013.2	/ Map / Offsets
2011 11 09	000000	02 10 40.0	+03 28 26	0.472	1.449	163.0	11.5	9.2	1.99	173.1	0	012.4	/ Map / Offsets
2011 11 15	000000	02 13 05.1	-00 41 10	0.525	1.482	155.1	16.3	9.6	1.53	170.0	0	011.7	/ Map / Offsets
2011 11 21	000000	02 15 49.6	-03 48 07	0.585	1.517	148.1	20.2	10.0	1.14	164.5	0	011.1	/ Map / Offsets

The trajectory in the sky is as below:



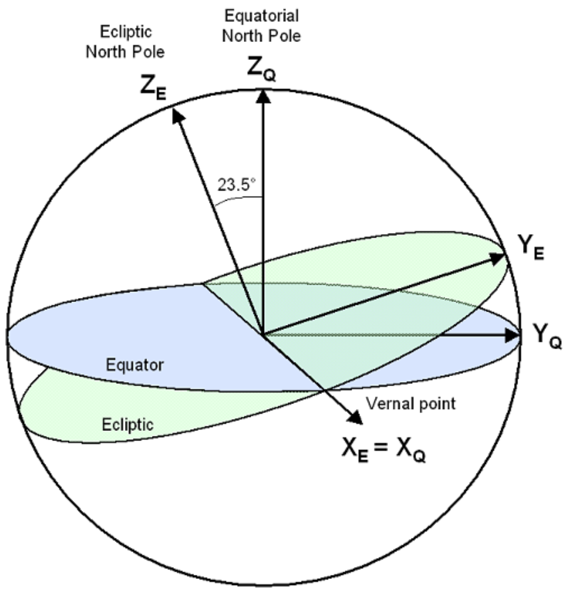
And the sky motion is also plotted below. It has a very strong maximum at mid-october.



2. Calculating the apparent rotation

2.1 Asteroid position

We define the following unit vectors:



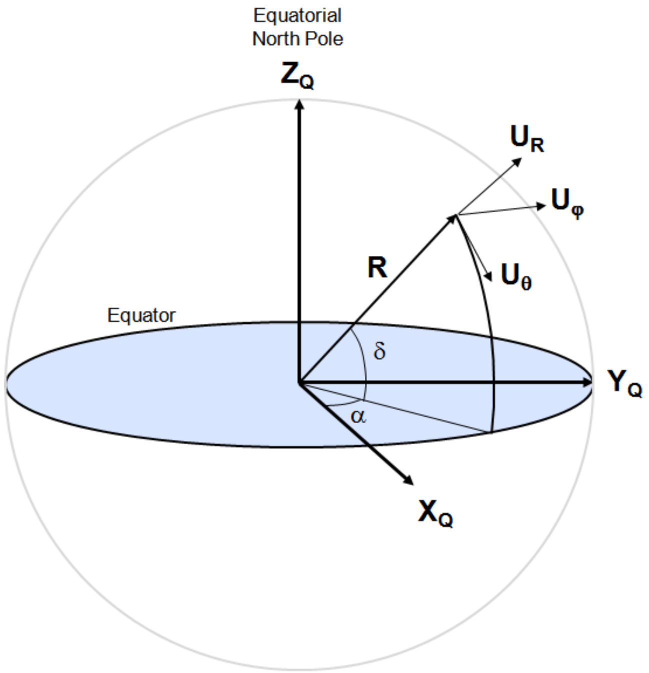
The MPC ephemeris provides the asteroid position with two angles: right ascension (α) and declination (δ).
We calculate the unit vector giving the asteroid direction from Earth center with

$$\mathbf{R} = \begin{pmatrix} \cos\alpha\cos\delta \\ \sin\alpha\cos\delta \\ \sin\delta \end{pmatrix}_{\mathbf{Q}} = \cos\alpha\cos\delta\,\mathbf{X_Q} + \sin\alpha\cos\delta\,\mathbf{Y_Q} + \sin\delta\,\mathbf{Z_Q}$$

The subscript Q reminds that the coordinates are relative to the equatorial axes ($\mathbf{X_Q}$,$\mathbf{Y_Q}$,$\mathbf{Z_Q}$).

2.2 Asteroid movement

We define three unit vectors ($\mathbf{U_R}$, **U**□, **U**□) as on the figure below:



They are given by

$$\mathbf{U_R} = \mathbf{R} = \begin{pmatrix} \cos\alpha\cos\delta \\ \sin\alpha\cos\delta \\ \sin\delta \end{pmatrix}_{\mathbf{Q}}$$

$$\mathbf{U_\theta} = \begin{pmatrix} \cos\alpha\sin\delta \\ \sin\alpha\sin\delta \\ -\cos\delta \end{pmatrix}_{\mathbf{Q}}$$

$$\mathbf{U}_\varphi = \begin{pmatrix} -\sin\alpha \\ \cos\alpha \\ 0 \end{pmatrix}_{\mathbf{Q}}$$

The MPC provides two quantities: the sky motion in arc sec per minutes ("/min), and the azimuth (P.A). We call then *V* and *Az*, respectively. When the azimuth is 0 deg, motion is along $-\mathbf{U}_\theta$; when the azimuth is 90 deg, motion is along $+\mathbf{U}_\varphi$. The apparent motion of the asteroid is described by the following vector

$$\mathbf{V} = V\left(-\mathbf{U_\theta}\cos Az + \mathbf{U}_\varphi \sin Az\right) = V\begin{pmatrix} -\cos Az\cos\alpha\sin\delta - \sin Az\sin\alpha \\ -\cos Az\sin\alpha\sin\delta + \sin Az\cos\alpha \\ \cos Az\cos\delta \end{pmatrix}_{\mathbf{Q}}$$

V has the same dimension as *V* and represents an angular speed (e.g. arcsec per minute).

2.3 Line of sight rotation

The velocity vector **V** corresponds to a rotation with the following rotation vector ω:

$$\boldsymbol{\omega} = \mathbf{R} \times \mathbf{V}$$

$$\boldsymbol{\omega} = -V\left(\mathbf{U}_{\boldsymbol{\theta}} \sin Az + \mathbf{U}_{\varphi} \cos Az\right) = -V \begin{pmatrix} \sin Az \cos\alpha \sin\delta - \cos Az \sin\alpha \\ \sin Az \sin\alpha \sin\delta + \cos Az \cos\alpha \\ -\sin Az \cos\delta \end{pmatrix}_{\mathbf{Q}}$$

The modulus of this vector gives the rotation speed, its direction gives the rotation axis.

2.4 Apparent rotation of the asteroid

Let's assume that the inertial rotation of the asteroid is described with a constant vector **Ω** (it is not "tumbling"). Then the apparent rotation is given by

$$\boldsymbol{\Omega}_{\mathbf{app}} = \boldsymbol{\Omega} + \boldsymbol{\omega}$$

With photometry we will measure the modulus of $\boldsymbol{\Omega}_{\mathbf{app}}$.

2.5 Results

We consider two cases : the inertial rotation vector points towards the North or South poles of the ecliptic. If ε is the ecliptic obliquity (~23.5 deg) then this means

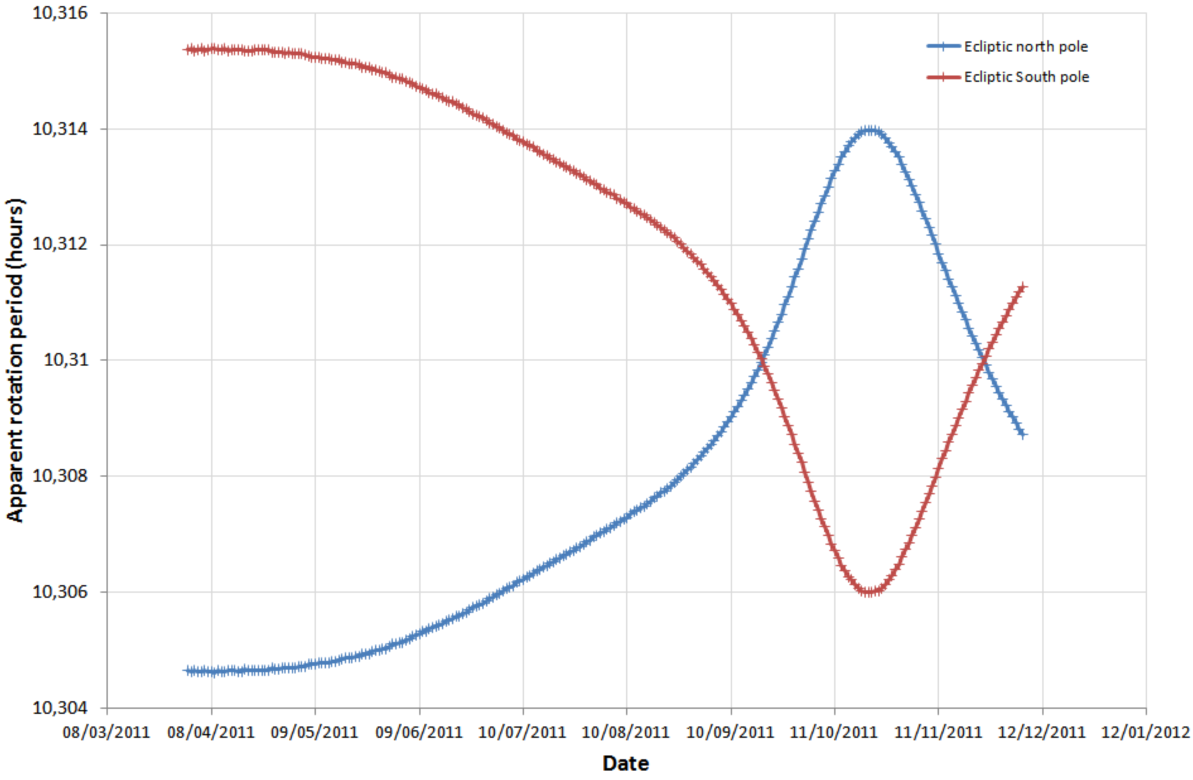
$$\boldsymbol{\Omega} = \pm\Omega \begin{pmatrix} 0 \\ -\sin\varepsilon \\ \cos\varepsilon \end{pmatrix}_{\mathbf{Q}}$$

Since the rotation period of 1036 Ganymed is about 10.31 hours, Ω is approximately equal to 2π/10.31 radians per hours.

Then we calculate the apparent rotation speed

$$\Omega_{app} = \left\| \pm\Omega \begin{pmatrix} 0 \\ -\sin\varepsilon \\ \cos\varepsilon \end{pmatrix}_{\mathbf{Q}} - V \begin{pmatrix} \sin Az \cos\alpha \sin\delta - \cos Az \sin\alpha \\ \sin Az \sin\alpha \sin\delta + \cos Az \cos\alpha \\ -\sin Az \cos\delta \end{pmatrix}_{\mathbf{Q}} \right\|$$

The results are plotted on the graph below.



3. Determination of the rotation axis

3.1 Problem

At the moment, it is clear that a careful measurement of the apparent rotation period in June-July 2011 and then in October 2011 should allow to discriminate between an exact prograde and an exact retrograde rotations, with the rotation axis pointing to one of the ecliptic poles.

But it might well be that the real rotation axis is none of these two extreme cases. If it has an inclination over the ecliptic, how can we measure it ? More generally, can we deduce from the measured variations of the apparent rotation speed the absolute orientation of the rotation axis ?

If we perform measurements of the apparent rotation period at two epochs 1 and 2, we will get the following two measurements

$$\Omega_{app}^{(1)} = \left\| \mathbf{\Omega} + \boldsymbol{\omega}_1 \right\|$$
$$\Omega_{app}^{(2)} = \left\| \mathbf{\Omega} + \boldsymbol{\omega}_2 \right\|$$

We know the two apparent rotation speeds (from the measurements), we know the rotation vectors $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ of the lines of sight (from the ephemeris). The only unknown is $\mathbf{\Omega}$, the vector of inertial rotation of the asteroid.

3.2 Solution

The problem can be reformulated by writing

$$\mathbf{V} = \mathbf{\Omega} + \boldsymbol{\omega}_1 = \begin{pmatrix} V_X \\ V_Y \\ V_Z \end{pmatrix}$$

$$\mathbf{dV} = \boldsymbol{\omega}_2 - \boldsymbol{\omega}_1 = \begin{pmatrix} dV_X \\ dV_Y \\ dV_Z \end{pmatrix}$$

Then we have

$$\Omega_{app}^{(1)} = \|\mathbf{V}\|$$
$$\Omega_{app}^{(2)} = \|\mathbf{V} + \mathbf{dV}\|$$

We know the two moduli $\|\mathbf{V}\|$ and $\|\mathbf{V} + \mathbf{dV}\|$ from the measurements, and we also know the components of the small vector $\mathbf{dV}$. To get more information about **V** we can express the variations of its modulus as a function of the perturbation **dV**:

$$\|\mathbf{V}\| = \sqrt{V_X^2 + V_Y^2 + V_Z^2}$$
$$\|\mathbf{V} + \mathbf{dV}\| = \|\mathbf{V}\| + \frac{\partial \|\mathbf{V}\|}{\partial V_X} dV_X + \frac{\partial \|\mathbf{V}\|}{\partial V_Y} dV_Y + \frac{\partial \|\mathbf{V}\|}{\partial V_Z} dV_Z$$

The term on the right-hand side is a gradient operator. We find

$$\frac{\partial \|\mathbf{V}\|}{\partial V_X} = \frac{V_X}{\sqrt{V_X^2 + V_Y^2 + V_Z^2}}$$

and similar expressions for the other derivatives. We get

$$\|\mathbf{V} + \mathbf{dV}\| = \|\mathbf{V}\| + \frac{\mathbf{V}}{\|\mathbf{V}\|} . \mathbf{dV}$$

Coming back to our original problem, this means

$$\Omega_{app}^{(2)} = \Omega_{app}^{(1)} + \frac{\mathbf{\Omega} + \boldsymbol{\omega}_1}{\|\mathbf{\Omega} + \boldsymbol{\omega}_1\|} . (\boldsymbol{\omega}_2 - \boldsymbol{\omega}_1)$$

Finally, since the perturbation due to the movement of the line of sight is very small we can write

$$\Omega_{app}^{(2)} \approx \Omega_{app}^{(1)} + \frac{\mathbf{\Omega}}{\|\mathbf{\Omega}\|} . (\boldsymbol{\omega}_2 - \boldsymbol{\omega}_1)$$

3.3 Discussion

The above equation means that the difference between the two measured apparent periods is equal to the product between the modulus of the vector ω_2-ω_1 and the cosine of the angle between **Ω** and ω_2-ω_1:

$$\Omega_{app}^{(2)} - \Omega_{app}^{(1)} = \left\| \boldsymbol{\omega}_2 - \boldsymbol{\omega}_1 \right\| \cos\left(\boldsymbol{\Omega}, \boldsymbol{\omega}_2 - \boldsymbol{\omega}_1\right)$$

Thus, to be in good conditions to observe variations of the apparent rotation period, the vector $\boldsymbol{\omega}_2$-$\boldsymbol{\omega}_1$ must have a large modulus. Then, the actually observed variations directly give the angle $\left(\boldsymbol{\Omega}, \boldsymbol{\omega}_2 - \boldsymbol{\omega}_1\right)$.

From this result, it is clear that the variation of Ω_{app} between two epochs only allows to get one angle, which is in general not sufficient to determinate the full rotation vector. If the cosine is close to +1 or -1 then the rotation axis is unambiguous. But in most cases, the rotation vector **Ω** is not fully determinated. It will point somewhere on a circular crown defined by all possible vectors making the calculated angle with the vector $\boldsymbol{\omega}_2$-$\boldsymbol{\omega}_1$.

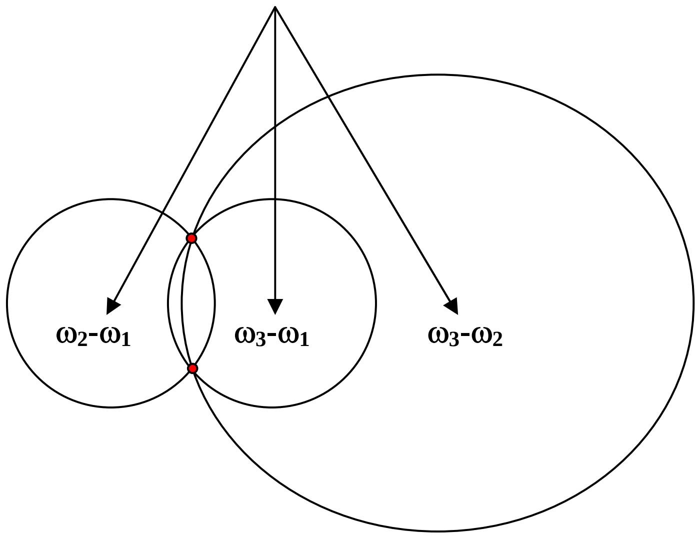
3.4 Combining 3 measurements or more

If now we combine 3 measurements

$$\Omega_{app}^{(1)} = \left\| \boldsymbol{\Omega} + \boldsymbol{\omega}_1 \right\|$$
$$\Omega_{app}^{(2)} = \left\| \boldsymbol{\Omega} + \boldsymbol{\omega}_2 \right\|$$
$$\Omega_{app}^{(3)} = \left\| \boldsymbol{\Omega} + \boldsymbol{\omega}_3 \right\|$$

using the method described previously, we will be able to calculate the three angles: $\left(\boldsymbol{\Omega}, \boldsymbol{\omega}_2 - \boldsymbol{\omega}_1\right)$, $\left(\boldsymbol{\Omega}, \boldsymbol{\omega}_3 - \boldsymbol{\omega}_2\right)$ and $\left(\boldsymbol{\Omega}, \boldsymbol{\omega}_3 - \boldsymbol{\omega}_1\right)$. Then for each case we have a circular crown, and the rotation axis will be defined by the intersection of the three crowns.

Unfortunately, it is generally not possible to determinate unambiguously the rotation axis with 3 measurements. The reason is that the three vectors $\left(\boldsymbol{\omega}_2 - \boldsymbol{\omega}_1\right)\left(\boldsymbol{\omega}_3 - \boldsymbol{\omega}_2\right)$ and $\left(\boldsymbol{\omega}_3 - \boldsymbol{\omega}_1\right)$ are linearly dependent and thus coplanar. Usually, we will obtain a situation like the one depicted below. The two possible solutions for the rotation axis (in red) are obtained by the intersections of the various crowns. The third combination does not allow to remove the ambiguity.



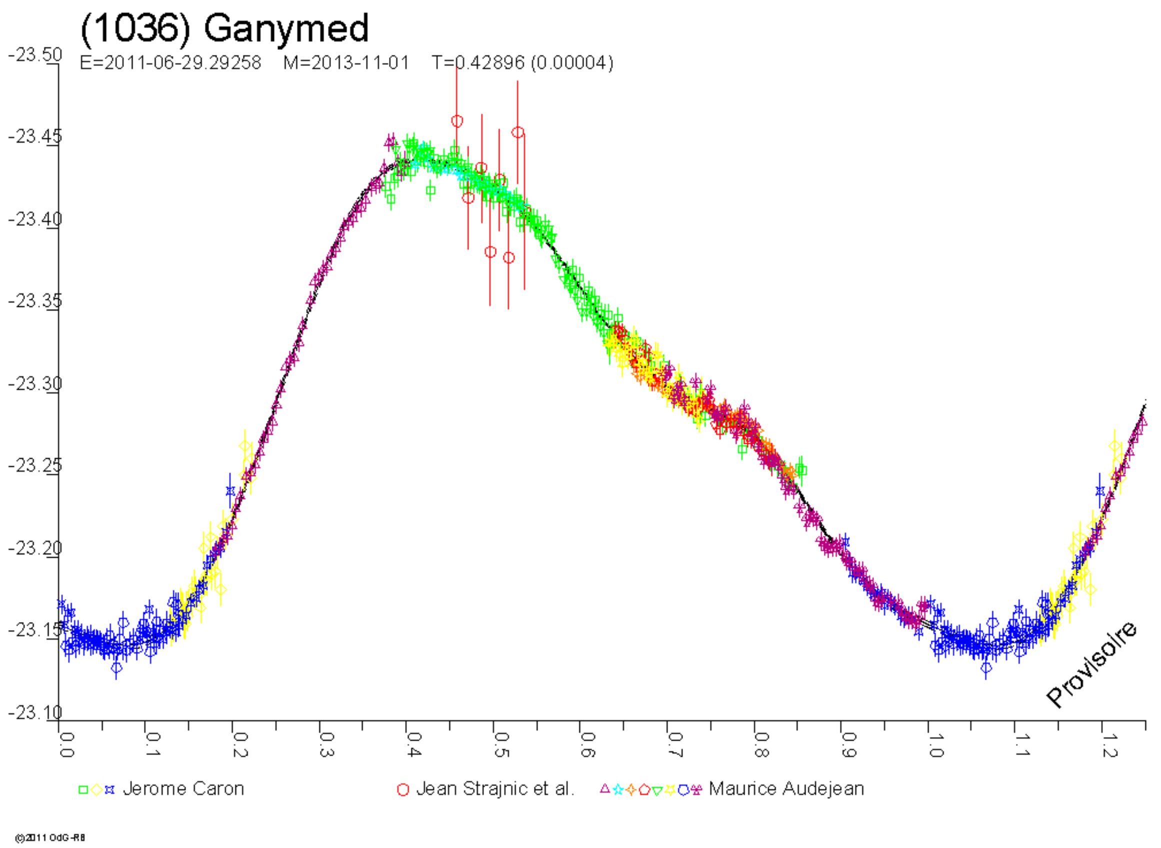
To have a complete, unambiguous determination of the rotation axis we would need 4 measurements of the apparent rotation speed, such that the three vectors $(\boldsymbol{\omega}_2 - \boldsymbol{\omega}_1)(\boldsymbol{\omega}_3 - \boldsymbol{\omega}_2)$ and $(\boldsymbol{\omega}_4 - \boldsymbol{\omega}_3)$ are not coplanar.

4. Preliminary results for Ganymed, and how to continue

4.1 Lightcurve and first period measurement

Ganymed has been observed between 21 June and 10 July 2011. The following lightcurve has been obtained.

Not all measurements are yet included (data processing in ongoing), and a few points measured by other observers in April 2011 have also been added. Due to the very low number of points, they should not impact the calculation of the apparent rotation period so that we can safely ignore them and consider that the measurements are all in the June-July period.



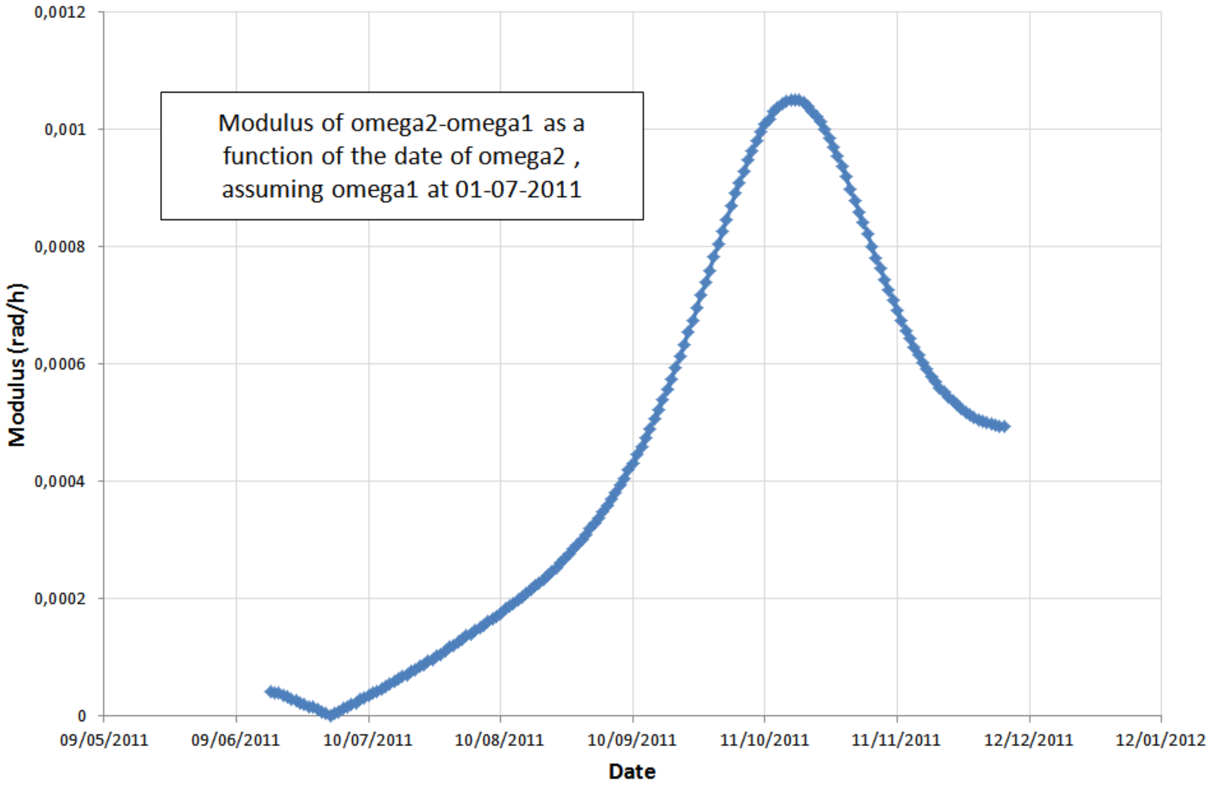
The lightcurve shows some relatively strong variations, which is a good thing as this will help to get accurate values for the period. So far, the relative accuracy obtained is 10^{-4} which seems sufficient to get information about the orientation of the rotation axis.

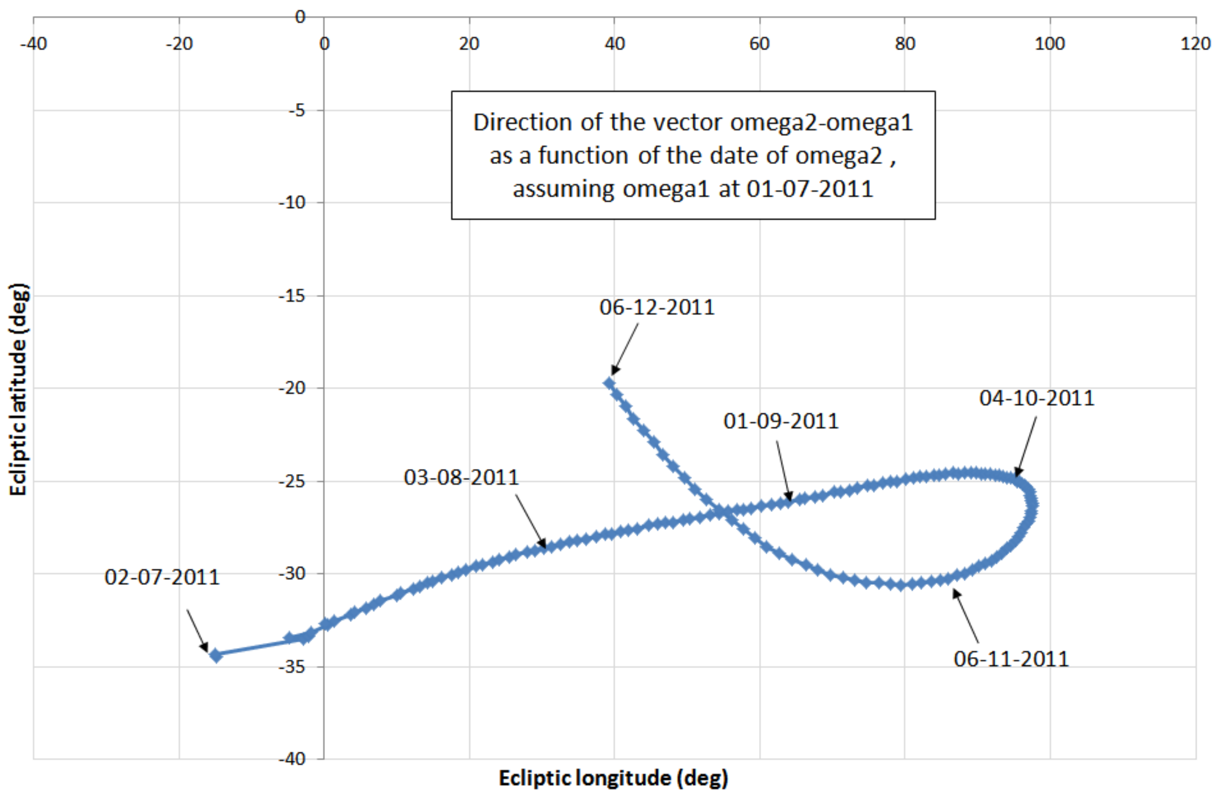
4.2 How to continue ?

Assuming that $\boldsymbol{\omega}_1$ was measured around 01 July, a second epoch (2) of measurement must be chosen so that the modulus $\|\boldsymbol{\omega}_2 - \boldsymbol{\omega}_1\|$ is maximized. Then the variations in Ω_{app} will be comfortably observed.

Now, if two epochs (2, 3) are chosen then not only the moduli $\|\boldsymbol{\omega}_2 - \boldsymbol{\omega}_1\|$ $\|\boldsymbol{\omega}_3 - \boldsymbol{\omega}_2\|$ and $\|\boldsymbol{\omega}_3 - \boldsymbol{\omega}_1\|$ must be maximized but also the directions of the vectors must differ significantly so that the angles that are measured, $(\boldsymbol{\Omega}, \boldsymbol{\omega}_2 - \boldsymbol{\omega}_1)$, $(\boldsymbol{\Omega}, \boldsymbol{\omega}_3 - \boldsymbol{\omega}_2)$ and $(\boldsymbol{\Omega}, \boldsymbol{\omega}_3 - \boldsymbol{\omega}_1)$ bring different constraints on the asteroid rotation vector $\boldsymbol{\Omega}$.

On the following graphs, the modulus and direction of $(\boldsymbol{\omega}_2 - \boldsymbol{\omega}_1)$are plotted as a function of the selected second epoch for measurements. The modulus must be compared to $\|\boldsymbol{\Omega}\|$ = 0.6094 rad/h. We see that end of October 2011, the ratio $\|\boldsymbol{\Omega}\| / \|\boldsymbol{\omega}_2 - \boldsymbol{\omega}_1\|$ reaches the value of 0.0017 at its maximum. The relative accuracy of the measurements of Ω_{app} must be better than this value which is already the case for the first measurements.





4.3 Conclusion

The best periods for one or two new measurements of the apparent rotation periods can be deduced from the above two plots. Two scenario can be proposed:

1. a single measurement of the apparent period could be started at the end of September 2011. Then the modulus $\left\| \boldsymbol{\omega}_2 - \boldsymbol{\omega}_1 \right\|$ is maximum so the measurement would be comfortable. Distinction can be made between exact prograde and retrograde rotations (with rotation axis pointing to one of the ecliptic poles), but most likely a large ambiguity would remain with a broad family of possible rotation vectors $\boldsymbol{\Omega}$.

2. two measurements of the apparent period could be made, one in the second half of August 2011 (because then $\left\| \boldsymbol{\omega}_2 - \boldsymbol{\omega}_1 \right\|$ becomes reasonably high), and the other in September-October 2011. Then the vectors $(\boldsymbol{\omega}_2 - \boldsymbol{\omega}_1)$ and $(\boldsymbol{\omega}_3 - \boldsymbol{\omega}_1)$ would be sufficiently different. Most likely, the rotation vector $\boldsymbol{\Omega}$ would be then almost fully determinated, with an ambiguity remaining between two possible solutions.